The Implementation of $\mu^-\mu^+$ Production by Muons in Geant4

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Abstract

When a high-energy muon interacts with a nucleus, a muon—anti-muon pair may be produced. This is known as muon pair production. It has a relatively low cross-section but becomes important in the regime of higher energies and is prominent when collecting high statistics. We implement it as a new electromagnetic process in Geant4. It is found that the muon pair production occurs 10^{-5} times less frequently than electron pair production at an energy of 100 TeV. This process will be useful for the LHC and FCC as the related experiments run at high energies and collect large statistics. The new process will be included in the next public release of Geant4.

Keywords

Geant4, Monte Carlo, muon, pair-production, LHC¶

Introduction

Geant4 is a toolkit for the Monte Carlo simulation of the passage of particles through matter [1]. It has wide applicability ranging from high energy physics, nuclear physics, space science, radiation medicine and others. Electromagnetic processes, i.e., processes involving gamma photons, electrons and muons, are a very important part of these simulations.

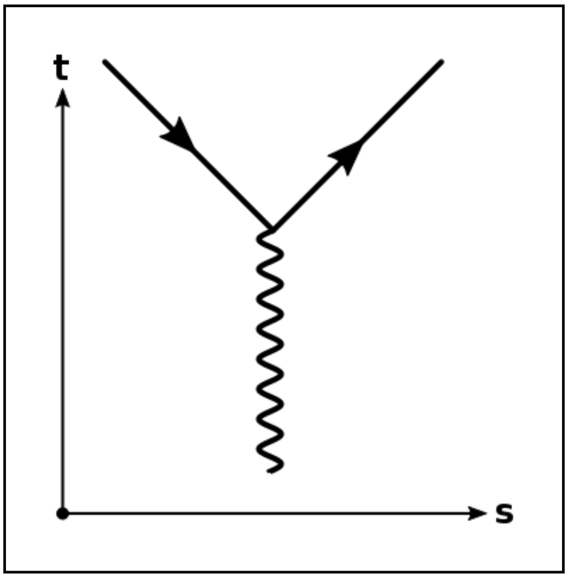
The muon is an essential particle in high energy physics (HEP) experiments as it passes through the detector centre and reaches the muon system. This allows us to reconstruct the tracks and identify details about the proton-proton collision. For example, the decay of a Z-boson to a muon pair has a very characteristic signature. In Geant4, muon simulations are of particular interest. At present, four muon processes are included in Geant4: ionisation, bremsstrahlung, electron-positron pair production and nuclear interactions.

In this report, we present the implementation of a new Geant4 process - muon to muon pair production. Although the process of muon pair production has a low cross-section, experiments at the LHC (and the FCC) collect very high statistics (order of billions of events). At these statistics, even low cross-section processes can affect the analyses of rare reaction channels.

One of the most detailed studies done on muon pair production was done in [3]. In the paper a study was done on muon-pair production by atmospheric muons in CosmoALEPH (ALEPH was a particle detector on the Large Electron-Positron Collider (LEP at CERN), CosmoALEPH refers to the run of ALEPH for cosmic ray detection).¶

Relevant Theory

Pair production refers to the process by which a lepton-anti-lepton pair is created from a virtual photon.



Feynman diagram for a basic pair production process

In our case, the process we are looking at is the pair production of muons. This can proceed through two channels. The first is direct muon pair production by the muon. The second is the bremsstrahlung of the muon followed by the production of a muon pair by the bremsstrahlung (virtual) photon of the muon.

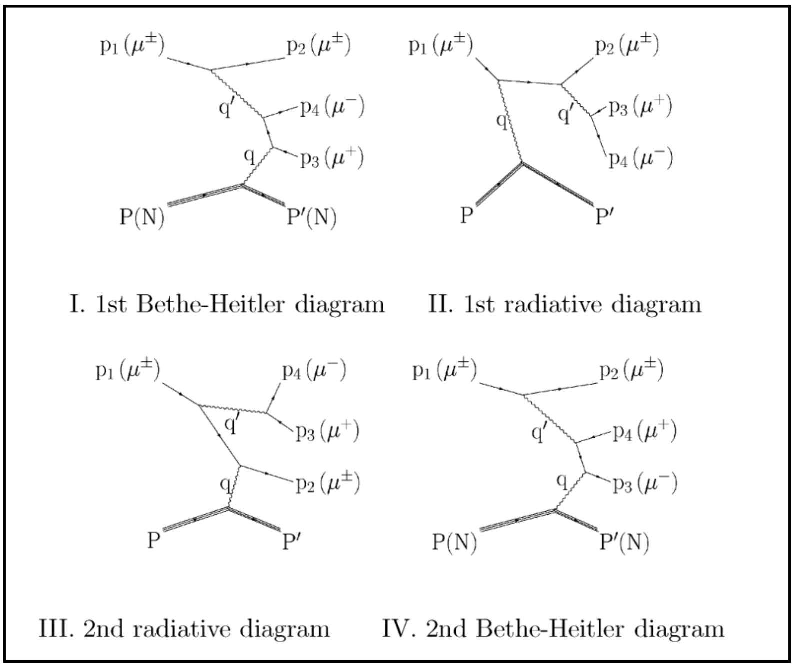


Figure 1 from [3]

Kelner studied the process of electron-positron pair production by muons in great depth in his paper [4]. A slightly modified version of his formula is used in Geant4 (G4MuPairProductionModel) to compute the cross section of electron-positron pair production by muons [7].

In the paper about muonic tridents, making a few modifications to the formula in [4], Kelner, et. al. arrived at a formula ((1) in [5]) describing the cross section of muon pair production by muons.

However, limitations of this formula were pointed out in a later paper. The limitations are as follows. Firstly, the approximation of a point-like nucleus is not appropriate. This is because the transferred momenta of the order of the muon mass are important and hence the nucleus cannot be approximated as a point-like Coulomb centre. Secondly, complete screening is used in the derivation. The regime of complete screening is approached very slowly with an increase in energy. Hence, the use of complete screening may result in the overestimation of the cross section.

In order to correct for these limitations, in the same paper, Kelner et. al. introduced a formula that takes into account the finite nucleus size as well as the screening effect at both low and high screening regimes. The formulae (15 - 21) from [6] approximate the cross section of the process to a very high accuracy. The formula is given by:

$$\sigma(E, v, \rho) dv d\rho = \frac{2}{3\pi}(Z\alpha r_\mu)^2 \frac{1-v}{v} \Phi(v, \rho) \ln(X) dv d\rho \quad -(1)$$

A description of the variables in the equation, the kinematic region and some other details can be found in Appendix A.¶

Code Implementation

In Geant4, there is a common design for all electromagnetic processes [8]. All models are implemented via an interface with G4VEmModel and all processes are implemented via an interface with G4VEnergyLossProcess. The model classes contain the details of how cross-section and related quantities are calculated. Process classes instantiate the model classes so that they can be used in simulations. For example, electron-positron pair production is implemented in G4MuPairProduction which instantiates G4MuPairProductionModel.

In our work, two new classes are created: a model class G4MuonToMuonPair-ProductionModel and a process class G4MuonToMuonPairProduction. These classes extend G4MuPairProductionModel and G4MuPairProduction respectively. G4MuPairProduction and G4MuonToMuonPairProduction are similar in most regards minus a few pedantic details such as which model is implemented. The major differences lie in the following two methods from G4MuPairProductionModel: SampleSecondaries whose task it is to create a distribution of energies, momenta and directions of the produced muons and ComputeDMicroscopicCrossSection which computes $d\sigma$. Another method U_func is added which is used to compute the function U as defined in [6].

ComputeDMicroscopicCrossSection computes the integral of (1) with respect to ρ using a setup very similar to the one defined in the physics reference manual page for positron-electron pair production by muons. The section of [7] - Comment on the Calculation of the Integral $\int d\rho$ - details the numerical integration procedure to be used. The limits of integration are changed in accordance with the the value of $\rho_{\max}$ defined in [6]. Apart from this, the only difference in the procedure is the formula of the cross section used. What this means for us is that we are using a tried, tested and optimised procedure rather than developing our own from scratch.

Modifications to TestEm17

In Geant4, the electromagnetic example TestEm17 serves as a means to test electromagnetic processes, specifically muon processes. After adding the new process into Geant4, a few modifications were necessary to be made in TestEm17. Firstly, PhysListEmStandard.cc was modified so that it included the new process. Next, MuCrossSections was modified so that it could compute the cross section for our new process in a way that could produce useful theoretical histograms like Figure 2. In fact, Figure 4 is the result of this modification. All the macro files for TestEm17 were edited so that they used the physics from PhysListEmStandard rather than from emstandard_opt0. We also edit RunAction, HistoManager and SteppingAction to account for the histograms for our new process. In total, we have added six new histograms: histograms for the simulated and theoretical values of $\log(E_{\text{loss}}/E_\mu)^*$ for our new process (Figures 3, 4); histograms for cross section vs energy of the incoming muon for four processes - ionisation, bremsstrahlung, electron pair production and muon pair production. These four histograms can be found in Appendix B.¶

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* E_{loss} is the energy lost by the primary muon and transferred to the muon pair

Results

The application and demonstration of the new process were done within the example TestEm17. The geometry of TestEm17 is a world which consists of a homogenous material (default: iron of thickness 1m). In the example, there are a few macro files to allow us to study the various muon processes. For our study, we use allproc.mac as the macro file. This macro includes all the muon processes except the muon nuclear interactions. The macro uses a beam of 10^6 primary muons each with an energy of 100 TeV.

To have a control setting, we initially disable the new process. Here are the results we get:

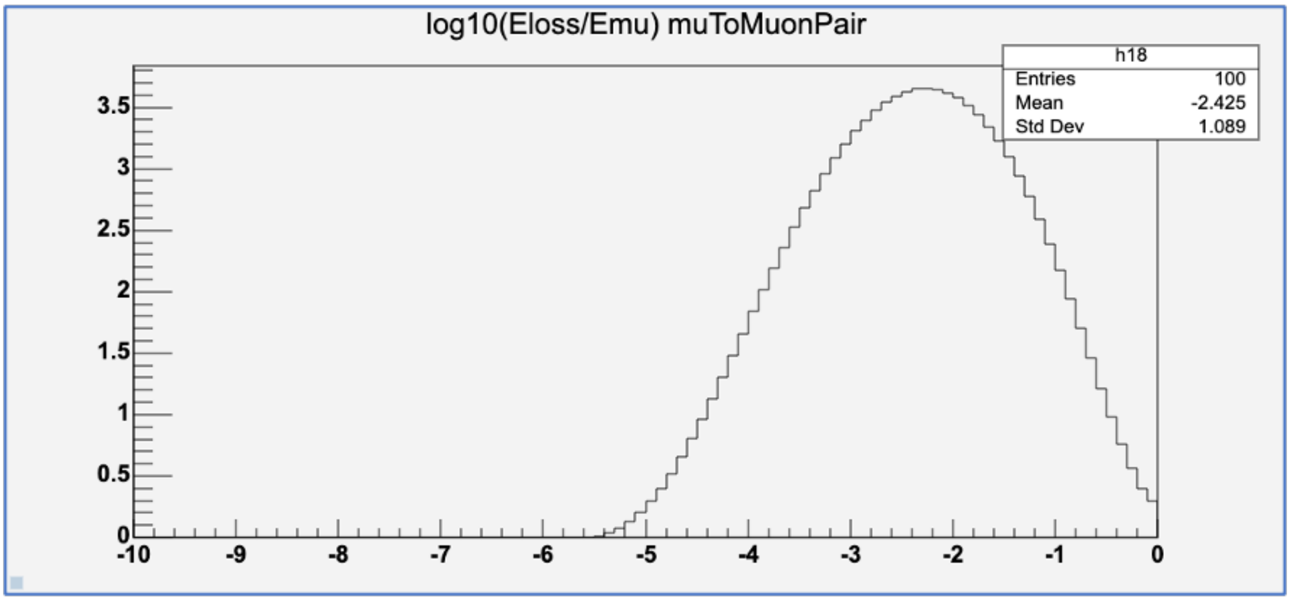
```
Run Summary
  Number of events processed : 1000000
  User=41.840000s Real=41.920882s Sys=0.080000s [Cpu=100.0%]

 The run consists of 1000000 mu+ of 1e+02 TeV through 1 m   of Iron (density: 7.9 g/cm3 )

 Number of process calls --->   muPairProd : 10540368   muIoni : 47204146       muBrems : 75376
 Simulation: total CrossSection = 0.5782 /cm     MeanFreePath = 1.7295 cm        massicCrossSection = 0.073469 cm2/g
 Theory:     total CrossSection = 0.58768 /cm    MeanFreePath = 1.7016 cm        massicCrossSection = 0.074673 cm2/g
```

And after enabling the process, these are the results we get:

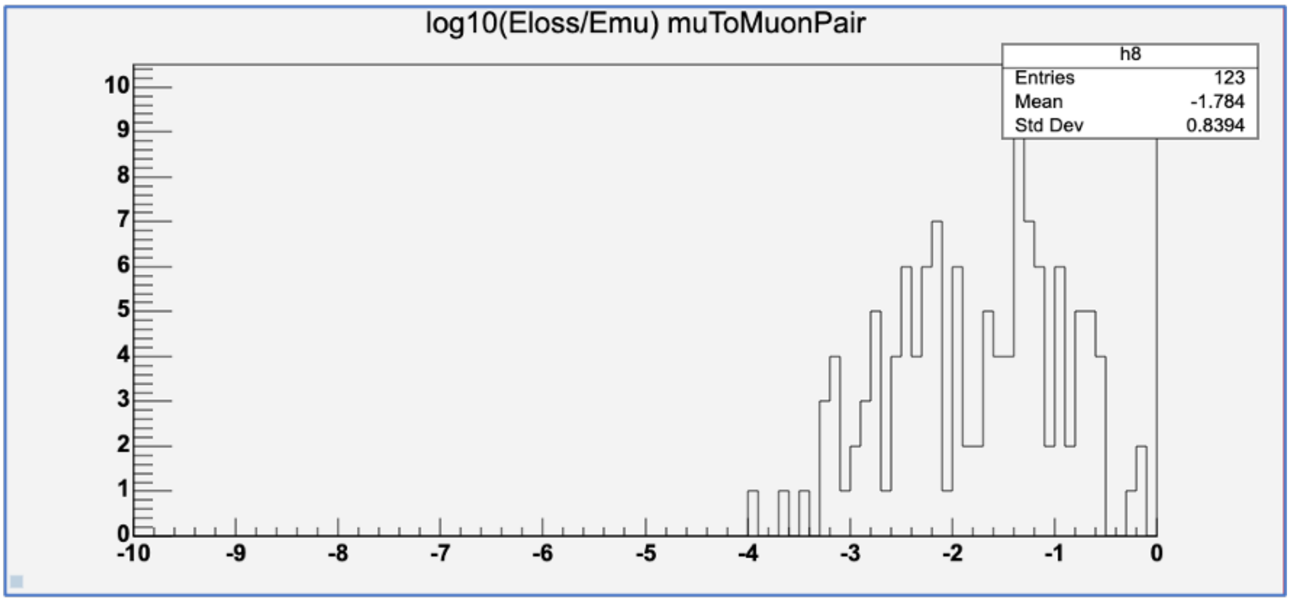
```
Run Summary
  Number of events processed : 1000000
  User=43.200000s Real=43.289958s Sys=0.080000s [Cpu=100.0%]

 The run consists of 1000000 mu+ of 1e+02 TeV through 1 m   of Iron (density: 7.9 g/cm3 )

 Number of process calls --->   muIoni : 47216477       muPairProd : 10540675   muBrems : 75473 muToMuonPairProd : 123
 Simulation: total CrossSection = 0.57833 /cm    MeanFreePath = 1.7291 cm        massicCrossSection = 0.073485 cm2/g
 Theory:     total CrossSection = 0.58768 /cm    MeanFreePath = 1.7016 cm        massicCrossSection = 0.074673 cm2/g
```

Note that the results of the two are quite similar; this is because the process of muon to muon-pair production has a relatively small cross section. We see that at 100 TeV, the number of muon pair production events is in the order of 10^{-5} the number of electron positron pair production events. Another subtlety is that due to the inclusion of our new process, we see a small increase in the number of each of the other events. This is to be expected since the muon is a secondary particle and is not killed off when it is produced. It is also produced with high enough energies to result in the other processes happening.

Below, we discuss a few histograms:

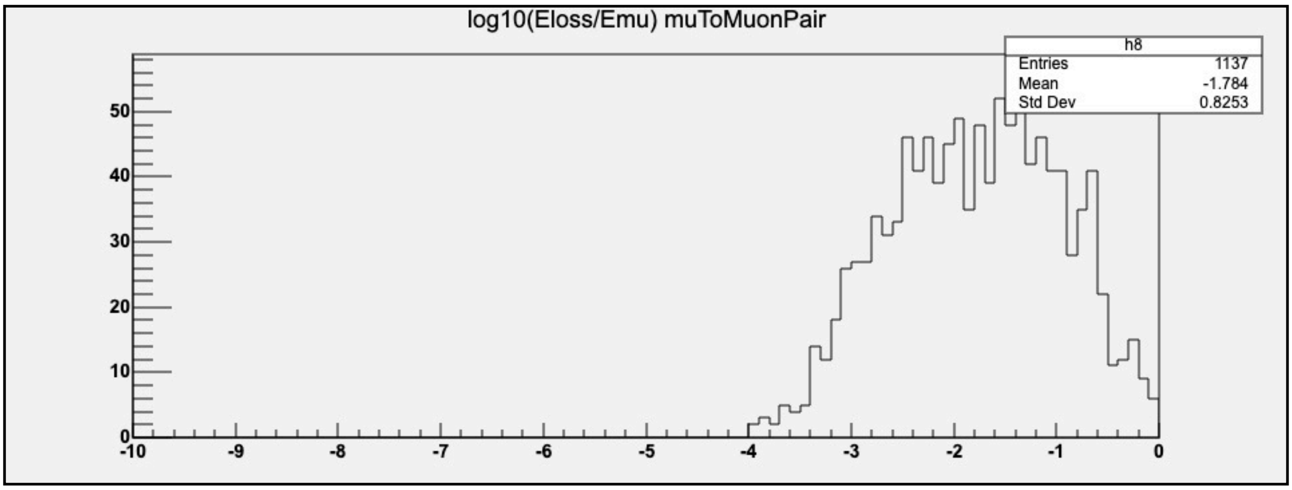


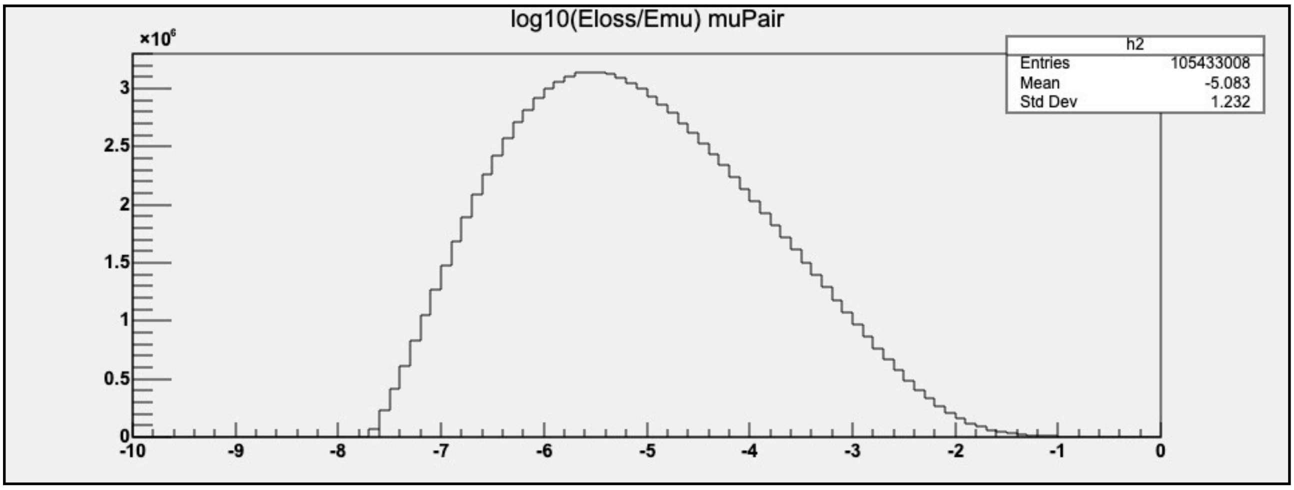
The above histogram shows the theoretical distribution of $\log(E_{\mathrm{loss}}/E_\mu)$ for muon to muon pair production. E_{loss} over here is the energy transferred to the muon pair from the primary particle and E_μ is the energy of the primary particle. We see that the mean energy transferred is 376 GeV.



The above histogram shows the simulated distribution of $\log(E_{\mathrm{loss}}/E_\mu)$ for the same process. Due to the relatively low statistics, we don't get a perfect histogram. The mean energy transferred is 1.64 TeV. We would need significantly higher statistics to approach the theoretical distribution.

It may also be of some use to compare the new process with electron-positron pair production. In order to do this, we generate histograms using higher statistics; 10^7 primary μ^+ to 10^6 primary μ^+.





We see a few things. Firstly, the theoretical and simulated histograms for the new process are have a similar shape. Further, the distribution for muon pair production is similar to that of electron-positron pair production. Secondly, the frequency of muon pair production events is 10^{-5} that of electron-positron pair production events. Lastly, the theoretical peak of energy transfer for muon pair production lies at 370 GeV which is quite a high energy resulting in other muon events happening. This is much greater than the mean energy of an electron-positron pair which is around 1 GeV.

The best way to validate the results would be to compare actual experimental data with the Monte Carlo simulation. Unfortunately, there isn't much experimental data which would allow us to confirm that our Monte Carlo simulation is correct. We can only rely on the fact that the theoretical formulae are correct.¶

Conclusion

The process of muon pair production is implemented in Geant4. The example TestEm17 has been updated to include the new process. These updates will be available in the next public release of Geant4. Results of simulations along with histograms are presented which correspond to theoretical predictions and qualitatively with the results of [3]. This aligns with our expectations based on the process of electron-positron pair production. For our given geometry and a primary muon of 100 TeV, we find that the ratio of muon pair production events to electron pair production events is of the order of 10^{-5}. We expect the process of muon pair production to be useful for analysis of rare processes at LHC and preparation for the FCC.¶

Appendix A: Theoretical cross section

As taken directly from [6], the theoretical cross section along with the details of all of the variables used is given as follows:

$$\sigma(E,v,\rho)\,dv\,d\rho = \frac{2}{3\pi}\,(Z\alpha\,r_\mu)^2\,\frac{1-v}{v}\,\Phi(v,\rho)\,\ln(X)\;dv\,d\rho\,. \tag{3}$$

Here $v = (E-E')/E = (E_+ + E_-)/E$ is the fraction of the energy transferred to the particles of the pair; $E,\ E'$ are initial and final energy of the parent particle; $E_+,\ E_-$ are energies of the produced particles; $\rho = (E_+ - E_-)/(E_+ + E_-)$ is the pair asymmetry parameter. Final energies are related with v, ρ by

$$E' = E\,(1-v)\,, \quad E_\pm = Ev\,(1\pm\rho)/2\,. \tag{4}$$

Kinematic region is defined by

$$2\mu/E \le v \le 1-\mu/E\,, \quad |\rho| \le \rho_{\max} \equiv 1 - 2\mu/(vE)\,. \tag{5}$$

Function Φ may be expressed as

$$\Phi(v,\rho) = \Big((2+\rho^2)(1+\beta) + \xi\,(3+\rho^2)\Big)\ln\left(1+\frac{1}{\xi}\right) - 1 - 3\,\rho^2 + \beta\,(1-2\,\rho^2) +$$

$$+\left((1+\rho^2)\left(1+\frac{3}{2}\beta\right) - \frac{1}{\xi}\,(1+2\,\beta)(1-\rho^2)\right)\ln(1+\xi)\,, \tag{6}$$

with

$$\xi = \frac{v^2(1-\rho^2)}{4\,(1-v)}\,, \quad \beta = \frac{v^2}{2\,(1-v)}\,. \tag{7}$$

Argument X of the logarithm in (3) is defined as follows. Let us denote as $U(E,v,\rho)$ the function

$$U(E,v,\rho) = \frac{0.65\,A^{-0.27}\,B\,Z^{-1/3}\,\mu/m}{1 + \dfrac{2\,\sqrt{e}\,\mu^2 B\,Z^{-1/3}(1+\xi)(1+Y)}{m\,E\,v\,(1-\rho^2)}}\,, \tag{8}$$

where $B = 183$, $e = 2.718\ldots$, A is atomic weight, $Y = 10\sqrt{\mu/E}$. Then

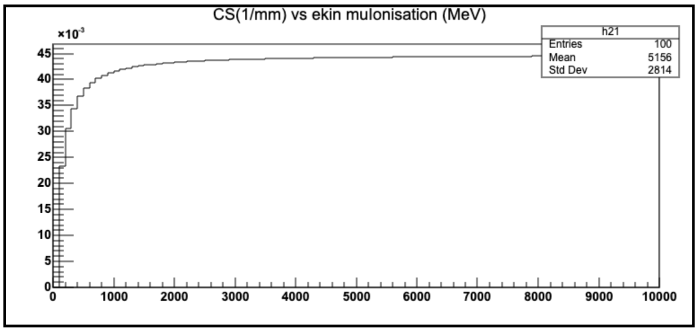
$$X = 1 + U(E,v,\rho) - U(E,v,\rho_{\max})\,, \tag{9}$$

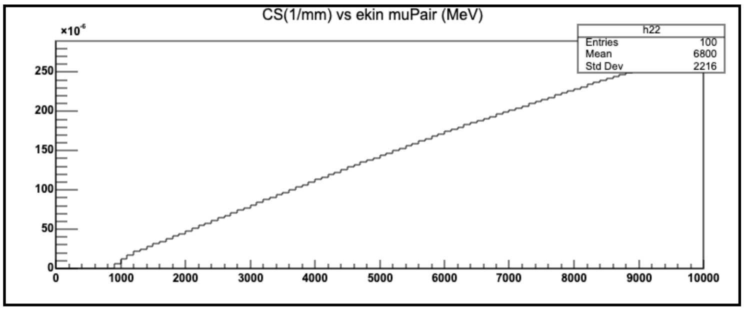
with $\rho_{\max}$ defined by (5). The function U is chosen in such a way to reproduce the main logarithmic factor in the limiting cases of absence of screening and complete screening. Function Y and numerical constants serve to improve the description of the total cross section. Cross section $\sigma(E,v,\rho)$ is non-negative in the kinematic region (5) and comes to zero at $\rho = \pm\rho_{\max}$. Comparison with numerical integration shows that the accuracy of (3) is better than 10% for $E > 10$ GeV and final particle energies E', $E_+, E_- > 1$ GeV, the total cross section being reproduced better than 3% for $E > 30$ GeV.

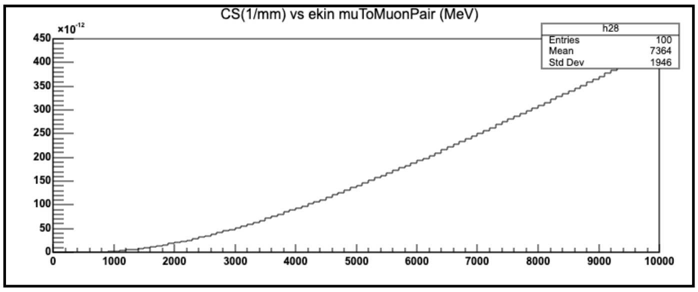
Formulae (3)-(9) describe the distribution of final particles in $(v,\ \rho)$ variables. To obtain the distribution in the energies of the particles of the pair $E_+,\ E_-$, it is sufficient to use

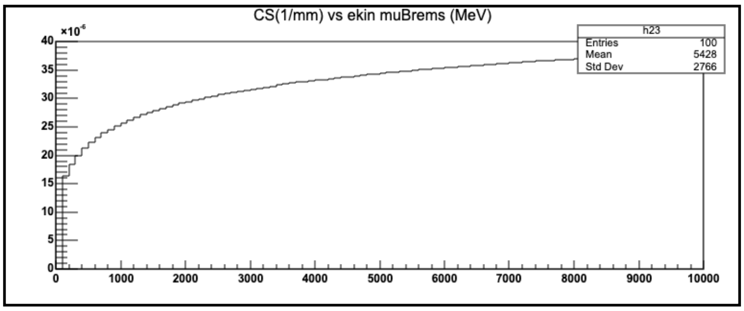
$$\sigma(E,E_+,E_-)\,dE_+\,dE_- = \frac{2}{E^2 v}\,\sigma(E,v,\rho)\,dE_+\,dE_-\,. \tag{10}$$

Appendix B: Additional Histograms









The above histograms represent the cross section (in mm^{-1}) of the four processes versus the primary energy (in MeV) of the particles. While the curves for ionisation and bremsstrahlung flatten out at higher energies, the curves for the two pair production processes increase almost linearly with energy.¶

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